APPLICATION OF CALCULUS

May – June 2021

QUESTION 10

10.1 The graph of $f(x) = ax^3 + bx^2 + cx + d$ has two turning points.

The following information about f is also given:

- $f(2) = 0$
- The x-axis is a tangent to the graph of f at $x = -1$
- $f'(1) = 0$
- $f'\left(\frac{1}{2}\right) > 0$

Without calculating the equation of f, use this information to draw a sketch graph of f, only indicating the x-coordinates of the x-intercepts and turning points. (4)

Given: $f(x) = 3x^3$

9.1 Solve $f(x) = f'(x)$ (3)

9.2 The graphs f, f' and f'' all pass through the point $(0\ ;\ 0)$.

9.2.1 For which of the graphs will $(0\ ;\ 0)$ be a stationary point? (1)

9.2.2 Explain the difference, if any, in the stationary points referred to in QUESTION 9.2.1. (2)

9.3 Determine the vertical distance between the graphs of f' and f'' at $x = 1$. (3)

9.4 For which value(s) of x is $f(x) - f'(x) < 0$? (4)

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9.2 If g is a cubic function with:

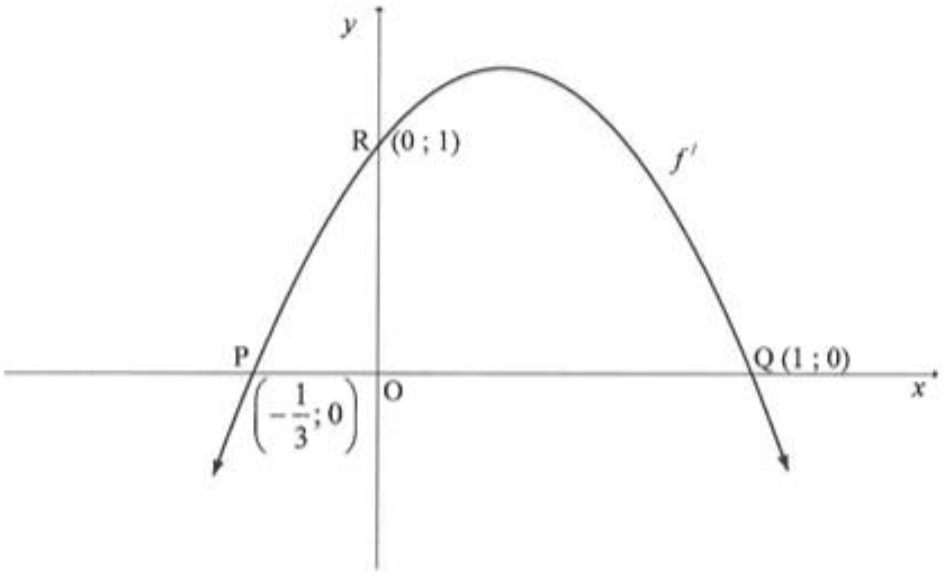
- $g(3) = g'(3) = 0$
- $g(0) = 27$
- $g''(x) > 0$ when $x < 3$ and $g''(x) < 0$ when $x > 3$,

draw a sketch graph of g indicating ALL relevant points. (3)

QUESTION 8

The graph of $y = f'(x) = mx^2 + nx + k$ is drawn below.

The graph passes the points $P\left(-\frac{1}{3}; 0\right)$, $Q(1; 0)$ and $R(0; 1)$.



8.1 Determine the values of m, n and k. (6)

8.2 If it is further given that $f(x) = -x^3 + x^2 + x + 2$:

8.2.1 Determine the coordinates of the turning points of f. (3)

8.2.2 Draw the graph of f. Indicate on your graph the coordinates of the turning points and the intercepts with the axes. (5)

8.3 Points E and W are two variable points on f' and are on the same horizontal line.

- h is a tangent to f' at E.
- g is a tangent to f' at W.
- h and g intersect at $D(a; b)$.

8.3.1 Write down the value of a. (1)

8.3.2 Determine the value(s) of b for which h and g will no longer be tangents to f'. (2)

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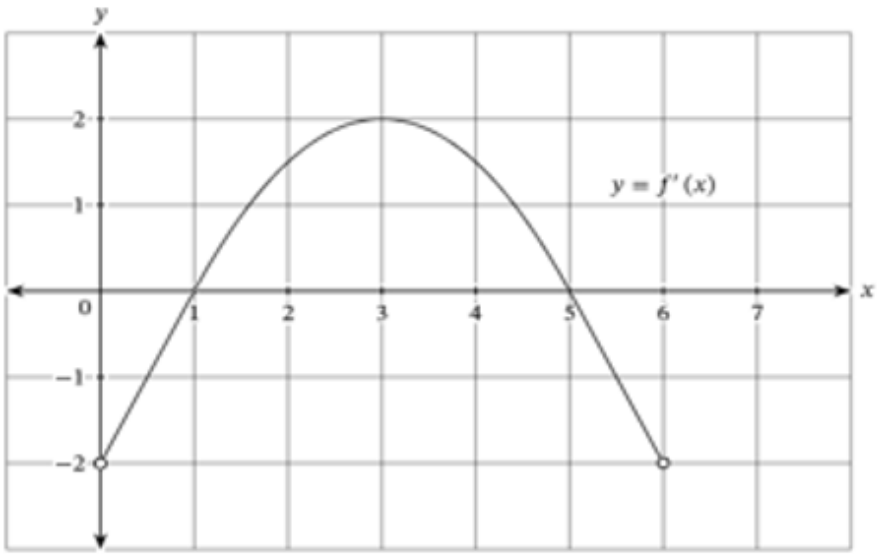
9.3 The graph of the derivative f' of a function f is shown.



9.3.1 Determine the x values at the turning points of the graph f. (2)

9.3.2 On what intervals is f decreasing? (4)